Measurement of Crystal Impedances at Low Levels*

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Summary—**It is very important to know the impedances of crystal diodes when constructing circuits such as mixers and detectors in which the crystals are used. It is always difficult to measure these impedances due to the nonlinear characteristics of the crystals but it is most difficult to make the measurements at minimum levels at which the crystals operate, since with such methods as the slotted line, the detector must operate at a still lower level to obtain the required probe decoupling. Thus, since the load whose impedance is being measured is itself a crystal operating at its minimum level, it is practically impossible to obtain a detector with sufficient sensitivity to make the measurement.**

Crystal impedances at these minimum levels are of utmost importance as it is here that optimum matching is essential for maximum sensitivity.

This paper describes practical techniques which use only standard equipment to measure crystal impedances at low levels. The detector used is a crystal of the same type as that being measured. The method is capable of precise results and good measurements can be obtained at low levels with little more effort than is normally required in making careful impedance measurements.

THE proper design of detectors and mixers using crystals requires a knowledge of the impedance characteristics of these crystals. Such knowledge is particularly important at low levels where only crystals are sensitive enough to serve as detectors or mixers and maximum sensitivity is required. It is difficult, however, to measure or even to define the impedance of a crystal due to its nonlinearity.

Consider the measurement of crystal impedances by the usual slotted line method in which the crystal whose impedance is to be measured is mounted in a suitable holder on the end of the line as a load (see Fig. 1).

Since the crystal is nonlinear its power level must be maintained constant throughout a measurement. The relative power level at which the crystal is operating can be determined by measuring its dc or modulation output, as shown in Fig. 1.

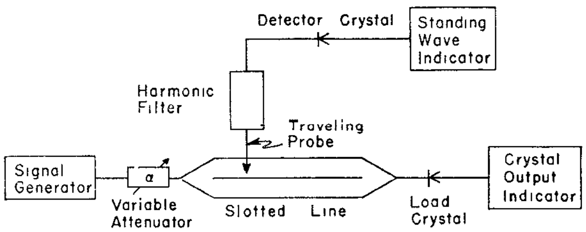

Fig. 1—Measurement of crystal impedances using modified slotted-line method.

Also when the measuring signal is applied to the crystal, harmonics are generated and fed back into the line, interfering with the measurements. The effect of these harmonics may be eliminated by a low-pass filter in the probe circuit to prevent them from reaching the detector, so that the measurements are made essentially at the fundamental frequency. The results are strictly valid only when the crystal is operating under conditions identical to those under which the measurements were made, but are useful as long as the output circuits are duplicated and low impedance return circuits are provided for the dc and modulation signals which are generated by the crystal.

Since it is desired to measure crystal impedance at the lowest levels at which they will operate, it is necessary to use a similar crystal as a detector, or as the mixer in a superheterodyne receiver. Thus the minimum level at which measurements can be made is determined by the detector crystal, which operates at a much lower level than the load crystal because of the necessary probe decoupling. Thus it is impossible to measure the impedances of crystals operating at very low levels by conventional slotted line methods.

Elimination of the difference in operating levels of the load and detector crystals resulting from the probe decoupling may be accomplished by interchanging the generator and detector as shown in Fig. 2. It can be

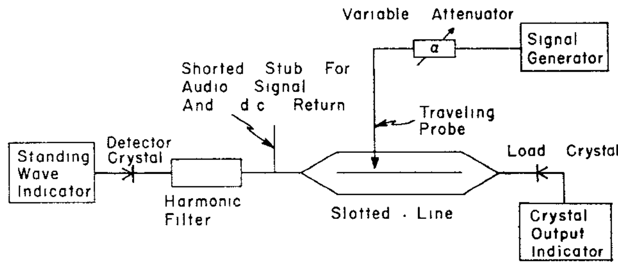

Fig. 2—Slotted-line measuring circuit obtained by interchanging generator and detector.

shown, (see Appendix I) that for loads on the ends of the slotted line, with a signal fed into the line through the probe, the current in one load as a function of probe position is proportional to the voltage existing in the line at the probe. The restrictions are the same as for the conventional slotted line method (*i.e.*, very loose coupling of the probe to the line, etc.).

It can be shown that if the two loads have the same standing wave ratio, there is a spacing for which the standing waves for the two loads will coincide, and the currents in the two loads will be identical at all times. Thus if the current in the load crystal is maintained at a fixed level, the current in the detector crystal will also remain constant. The voltage standing wave ratio will be determined from the variation in attenuation required to maintain the level constant. Thus the system is linear and, with an accurately calibrated at-

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tenuator, crystal impedances can be precisely measured at low levels.

The two standing waves in the line can be made to coincide quite closely, by means of a triple-stub tuner in the detector circuit as shown in Fig. 3. This tuner also serves as a dc and modulation signal return for both crystals. The fact that, in practice, the two standing waves in the line are not likely to coincide exactly means that there is some variation in the detector crystal output. This variation must be added to the attenuator variation to obtain the voltage standing-wave ratio induced in the line by the load crystal. With careful tuning this variation can be kept small so that the nonlinear characteristics of the detector crystal do not affect the system appreciably.

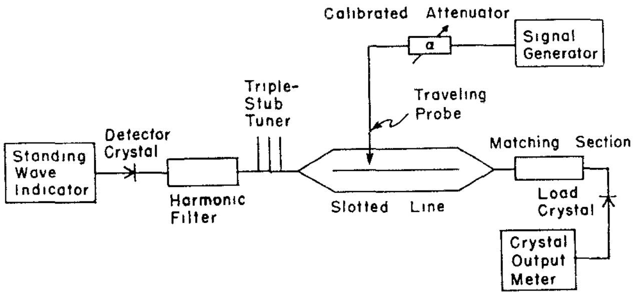

Fig. 3—Circuit for measuring crystal impedances at low levels.

At low levels the impedance of the load crystal is so high that the effective impedance at a high-voltage point in the line is comparable to the transfer impedance between the probe and the line. When this occurs the probe is no longer loosely coupled to the line and consequently the measurements are no longer valid.

However, a matching section may be used to transform the high impedance of the crystal to a value more nearly equal to the characteristic impedance of the line, thus reducing the standing waves in the line. This method requires an accurately constructed matching section and involves considerable computation to determine the impedance of the crystal itself. The matching section may introduce considerable loss which can interfere with the measurement of the crystal. However, the crystal most certainly will be used with a matching section in practice, in which case the losses in the matching section should logically be included in the measurements.

The high crystal impedances may also be determined without a matching section by making all measurements in the vicinity of a voltage minimum[1] (see Appendix II). Such measurements require very precise positioning of the probe for accuracy.

Curve 1 in Fig. 4 shows the voltage standing-wave ratio of a crystal alone with respect to a 50 ohm line. The measurements were made with a matching network as shown in Fig. 3, and then the impedance of the crystal itself was calculated from these measurements.

[1] Montgomery, "Technique of microwave measurements," McGraw-Hill Book Company, New York, p. 505, 1947.

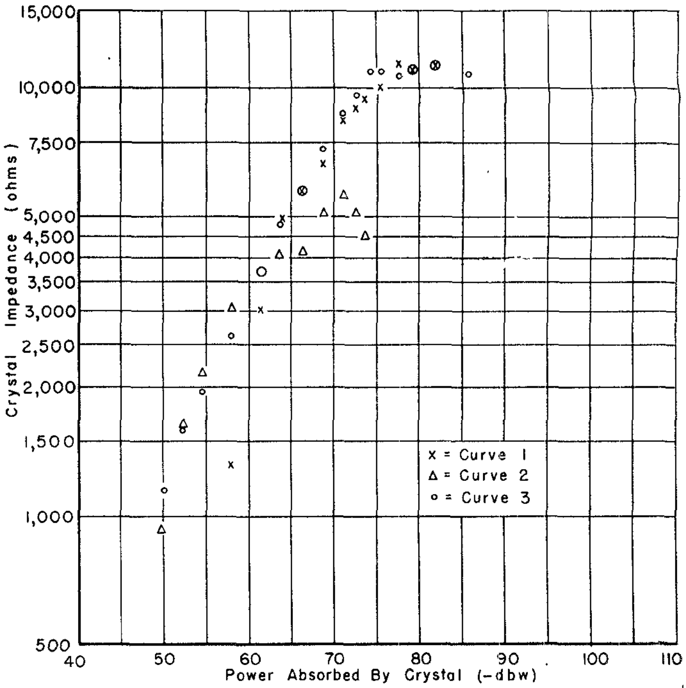

Fig. 4—Comparison of voltage standing-wave ratios determined by three different measurement techniques.

Curve 2 shows the measurements made on the same crystal with no matching section.

Curve 3 is obtained without the matching section but with all measurements made in the vicinity of a voltage minimum.

The Curves in Fig. 5 show the voltage standing-wave ratios of a number of 1N21B crystals with respect to 50 ohms, as determined by measurements made in the vicinity of a voltage minimum.

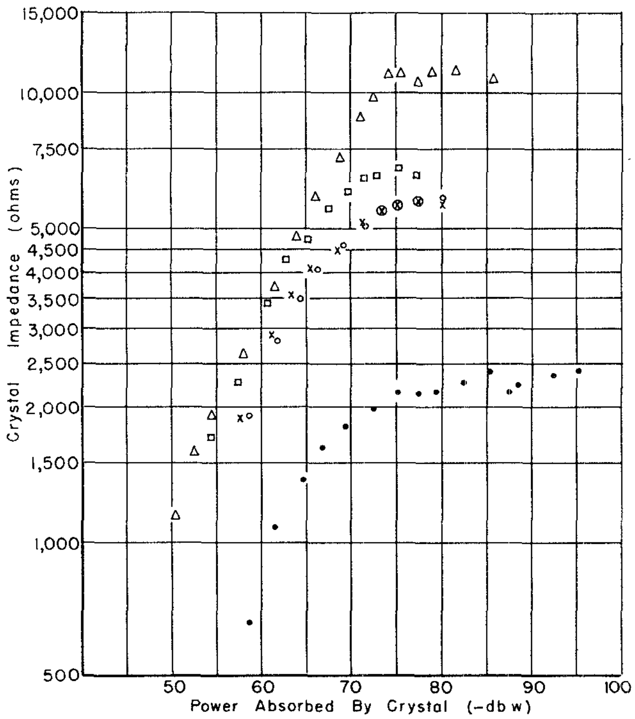

Fig. 5—Voltage standing-wave ratios with respect to fifty ohms measured on five different 1N21B crystals.

Conclusion

Curves 1 and 3 in Fig. 4 inspire considerable confidence in the two methods of measuring crystal impedances at low levels.

The techniques described should be valuable in obtaining crystal impedance data as the basis for the design of crystal mixers and detectors, for further studying the characteristics of crystals, and, as suggested by Fig. 5, for selecting crystals with similar characteristics.

Appendix I

Consider a slotted line with a load on each end which is fed through a loosely coupled probe as shown in the diagram of Fig. 6(a). Figure 6(b) is the equivalent circuit diagram. It is desired to know the currents in the impedances as a function of probe position.

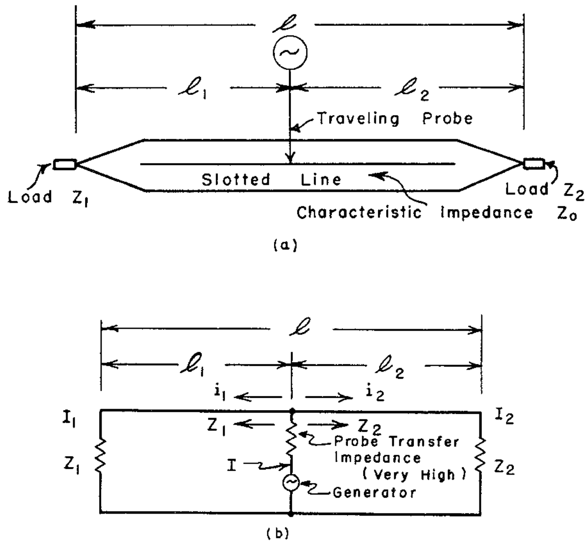

Fig. 6—(a) Diagram of the slotted-line setup. (b) Equivalent circuit diagram.

All impedances are normalized to the characteristic impedance Z_0 of the slotted line. Since the probe is loosely coupled to the line the transfer impedance is very high and the probe current I is considered to be constant.

i_1 and z_1 respectively are the current and the impedance seen by the probe looking toward Z_1. Similarly i_2 and z_2 are the current and impedance respectively, seen by the probe looking toward Z_2. Now by conventional transmission line theory:

$$i_1 = I_1(\cos \beta l_1 + Z_1 \sin \beta l_1)$$

$$i_2 = I_2(\cos \beta l_2 + jZ_2 \sin \beta l_2)$$

$$z_1 = \frac{Z_1 \cos \beta l_1 + j \sin \beta l_1}{\cos \beta l_1 + jZ_1 \sin \beta l_1}$$

$$z_2 = \frac{Z_2 \cos \beta l_2 + j \sin \beta l_2}{\cos \beta l_2 + jZ_2 \sin \beta l_2}.$$

At the probe the voltages across the two branches are equal, *i.e.*:

$$i_1 z_1 = i_2 z_2.$$

Solving for I_2,

$$I_2 = I_1 \frac{Z_1 \cos \beta l_1 + j \sin \beta l_1}{Z_2 \cos \beta l_2 + j \sin \beta l_2}.$$

The total probe current I will be the sum of the two branch currents, *i.e.*,

$$I = i_1 + i_2.$$

Solving these equations for I_1:

$$I_1 = \frac{I}{(Z_1+Z_2) \cos \beta l + j(1+Z_1Z_2) \sin \beta l} (Z_2 \cos \beta l_2 + j \sin \beta l_2)$$

$$= i(Z_2 \cos \beta l_2 + j \sin \beta l_2)$$

where

$$i = \frac{I}{(Z_1 + Z_2) \cos \beta l + j(1 + Z_1 Z_2) \sin \beta l}$$

is a constant since I is assumed to be constant and Z_1, Z_2, and l are fixed. Hence

$$I_1 = \alpha(Z_2 \cos \beta l + j \sin \beta l),$$

which is precisely the same variation as is experienced by the voltage in a line of characteristic impedance Z_0 when it is terminated in an impedance of Z_2. Thus the voltage wave in the slotted line due to the load Z_2 may be determined by the current in Z_1. Similarly the voltage wave in the line due to the load Z_1 may be measured by the current in Z_2.

Appendix II

It is to be noted that if the two standing waves in the line do not coincide exactly it will be inconvenient to apply the so-called "twice-minimum" technique, since two values, the detector variation and the attenuator variation, are involved. However, measurements at two arbitrary points near the minimum give the same results. The following formula is convenient for determining the resulting voltage standing-wave ratio:

$$\text{vswr} \cong \frac{\sqrt{R_1 - 1} + \sqrt{R_2 - 1}}{2\pi d_\lambda} \quad \text{for } d_\lambda \text{ small}$$

where

$$R_1 = \left| \frac{E_1}{E_m} \right|^2 \quad \text{and} \quad R_2 = \left| \frac{E_2}{E_m} \right|^2.$$

E_1, E_2, and E_m are voltages measured at points l_1, l_2, and the minimum respectively, and d_0 is the distance in wavelengths between the points l_1 and l_2.